Binding Energy and the Energy Released in a Fusion Reaction

The goal of nuclear reactions is to produce a more stable nucleus. In terms of binding energy per nucleon, this means a greater value should be obtained. On the other hand, we know that nuclear reactions create a great deal of energy. How does this all work?

A simple example:

Two deuterium nuclei fuse to produce an isotope of Helium and a neutron according to the following reaction.

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + {}^{1}_{0}n$$

We can calculate the binding energy per nucleon for deuterium:

The deuterium nucleus consists of 1 proton and 1 neutron. The combined mass of the two constituent particles = 1.007276 u + 1.008665 u = 2.015941 uThe actual mass of the deuterium nucleus = atomic mass – $m_e = 2.014102 \text{ u} - 0.000549 \text{ u} = 2.013553 \text{ u}$

This gives us a mass defect, $\Delta m = 2.015941 - 2.013553 = 0.002388 \text{ u} = 2.22 \text{ MeV c}^{-2}$

So, the binding energy per nucleon = 2.22 MeV / 2 = 1.11 MeV / nucleon

We can calculate the binding energy per nucleon for He-3:

The nucleus consists of 2 protons and 1 neutron. The combined mass = 2(1.007276) + 1.008665 = 3.023217 uThe actual mass of He-3 nucleus = atomic mass $- 2m_e = 3.016029 - 2(0.000549) = 3.014931 \text{ u}$

The mass defect = $3.023217 - 3.014931 = 0.008286 \text{ u} = 7.72 \text{ MeV c}^{-2}$

So, the binding energy per nucleon = 7.72 MeV / 3 = 2.57 MeV / nucleon

The product (He-3) has a greater binding energy per nucleon than the reactants (H-2).

On the other hand, if we look at the mass of the reactants (parents) and the mass of the products (daughters) we find something very interesting.

$$\begin{split} M_{parents} &= 2(2.013553 \text{ u}) = 4.027106 \text{ u} \\ M_{daughters} &= 3.014931 + 1.008665 = 4.023596 \text{ u} \end{split}$$

Notice that the daughters have less mass than the parents. This means that during the fusion reaction, we have also converted some mass into energy. The actual energy given off during the reaction is found using $E = \Delta m c^2$. In this case:

 $\Delta m = M_{parents} - M_{daughters} = 4.027106 - 4.023596 = 0.00351 \ u = 3.27 \ MeV \ c^{-2}$

The reaction releases = 3.27 MeV. This shows up as the kinetic energy of the daughters (products).